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The Concept of Angles Formed by a Transversal: A Study of Undergraduate Students' Learned Knowledge in the Didactic Transposition Process

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ABSTRACT

Didactic transposition is the process of transposing knowledge from reconstructed knowledge to knowledge that is organized and taught in the classroom. This method is critical to ensuring that there are no gaps or misconceptions between student knowledge and scholarly knowledge. Given the significance of didactic transposition, particularly of learned knowledge, a qualitative study using phenomenology as the design was carried out to ascertain the concept images of undergraduate students. This study focused on how undergraduate students conceptualized the concept of angles formed by a transversal. Nine undergraduate students in one of the public university in Aceh were given two questions. The results showed that six students had formed an incorrect concept image. They believed that pairs of angles would be formed if two parallel lines were cut by a transversal; each pair of corresponding angles, alternate interior angles, and alternate exterior angles must be congruent; and the measure of the angles must be known, to form each pair of same-side angles. These results indicate that there was an epistemological learning obstacle. Based on these findings, it is recommended that more processes of didactic transposition be revealed to determine the origins of these obstacles. Thus, alternative learning designs can be created to overcome this problem.

Keywords: angles formed by a transversal, concept image, didactic transposition, geometry, learned knowledge.

1. Introduction

A framework for analyzing the process of transposing scholarly knowledge to material studied by students,which he called didactical transposition (see Figure 1). Chevallard (1989, 2019) stated that didactic transposition was transpositional knowledge between mathematical knowledge that was produced by mathematicians (scholarly knowledge), mathematics that must be taught based on the curriculum (knowledge to be taught), knowledge of mathematics that was taught (taught knowledge), and knowledge of mathematics that was learned (learned knowledge). Didactic transposition is a model for understanding didactic phenomena, including what knowledge teachers teach, how the education system reconstructs knowledge, and how mathematicians develop knowledge (Putra, 2020).

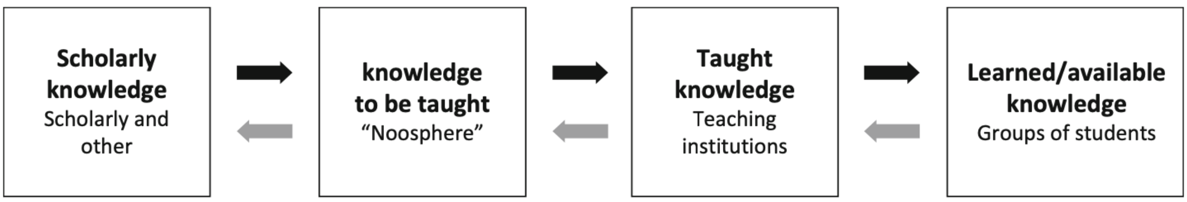

Figure 1. Didactic Transposition Process (Bosch & Gascón, 2006; Chevallard & Bosch, 2020)

Didactic transposition is important to ensure that there are no misconceptions or gaps between the knowledge learned by students and the knowledge produced by mathematicians. A good didactic transposition process will impact on providing right mathematical concepts, and the formation of appropriate learning situations (Jamilah et al., 2020).

The last step of didactic transposition is learned knowledge, knowledge as it is actually learned by students or acquired by learners (Bosch & Gascón, 2006). The learned knowledge can form a conception of the students about a topic. The conceptions created in pupils' brains by a series of instances and the characteristics of the examples given to them are known as concept image (Vinner, 1983, 2020). It is imperative to ensure that the concepts learned by the students are right and understandable since a lack of understanding of a concept caused fail in solving mathematical problems (Abdullah et al., 2015; Herizal et al., 2019; Sulastri et al., 2021).

In the learning process, the teacher designs a series of tasks to construct a concept. On other occasions, students may read concepts from books. Usually, the students will try to build their own version of the concept. The concept image that is formed has the possibility of being different from the actual concept definition. If that happens, then learning obstacles arise, for example, epistemological obstacles. Therefore, revealing students' conceptual images becomes very important for teaching. This not only provides a better understanding of students (knowing what causes them to act the way they do) but may also suggest some improvements to teaching that form an incorrect concept image (Vinner, 1983).

One of the fundamental principles in geometry is the concept of angles formed by a transversal. This concept is part of the parallelism concept. Several studies have shown that there is a problem related to these concepts. Baidoo & Baidoo (2022) found that new preservice mathematics teachers have lack understanding about the concept of corresponding angles and alternate angles. So far, research on this concept has emphasized its use in problem-solving, such as research by Baidoo & Baidoo (2022); Ekayanti (2017); Tupulu (2022); and Udiyono & Yuwono (2019). There is limited research focused on students’ concept images. Due to the importance of both the concepts of angles formed by a transversal and the disclosure of the concept image, a study was conducted. The objective of the study was to ascertain the concept images of undergraduate students. This study focused on how undergraduate students conceptualized the concept of angles formed by a transversal.

2. RESEARCH METHOD

A qualitative study with phenomenology as the design was conducted to uncover undergraduate students’ learned knowledge of the concept of angles formed by a transversal. The use of qualitative study because it is thought to provide an in-depth description of a specific program, activity, or setting (Mertens, 2010). Furthermore, phenomenology was chosen as the research design method because phenomenological research is a design in which the researcher to describe the participants' lived experiences with a phenomenon (Giorgi, 2009; Moustakas, 1994).

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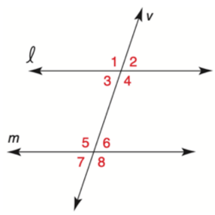
1. Given line $l \parallel m$ and a transversal v as seen in the figure. If any, Determine:
   a. All pairs of corresponding angles,
   b. All pairs of alternate interior angles,
   c. All pairs of alternate exterior angles, and
   d. All pairs of same-side angles

   If each pair of angles does not exist, give a reason!



2. Given line $r \nparallel s$ and a transversal t as seen in the figure below. If any, Determine:
   a. All pairs of corresponding angles,
   b. All pairs of alternate interior angles,
   c. All pairs of alternate exterior angles, and
   d. All pairs of same-side angles

   If each pair of angles does not exist, give a reason!

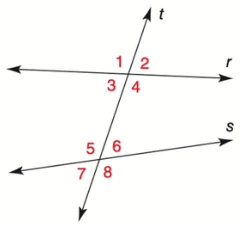


Figure 2. Questions for the test

The participants were nine preservice mathematics teacher students in one of public university in Aceh, Indonesia. To uncover their concept images, two questions were given to them (See Figure 2). The problems focused on the concept of angles formed by a transversal. Data were analyzed using Miles and Huberman step, i.e. Data reduction, data display, and drawing conclusion (Creswell, 2014).

3. RESULTS AND DISCUSSION

As previously stated, this study focused on how undergraduate students comprehend angles formed by a transversal. Two questions were posed to them to elicit this idea. The first question challenged students to find each pair of corresponding angles, alternate interior angles, alternate exterior angles, and same-side angles where the condition given is two parallel lines cut by a transversal. This problem was correctly solved by all of the participants. This indicates that they understand which one is a pair of corresponding angles, and so on. Overall, there were no issues with question 1.

In question number 2, the problem was the same as the previous question, namely, if determine each pair of corresponding angles, alternate interior angles, alternate exterior angles, and same-side angles, if any. However, the condition given was different. The criteria in this question were any two nonparallel lines cut by a transversal. The responses of the students revealed something unexpected. Six of the nine participants responded that no pair of angles were formed (see Figure 3 for an example of the answer), while the rest answered correctly. They answered that if any two nonparallel lines are cut by a transversal, no pair of corresponding angles, alternate interior angles, alternate exterior angles, and same-side angles are formed.

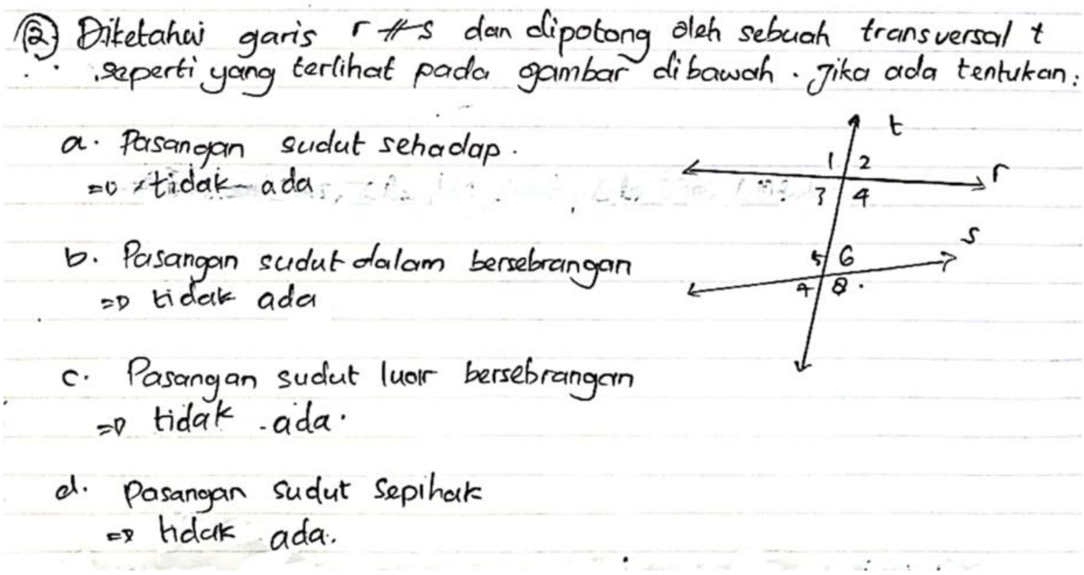

(2) Diketahui garis r ∦ s dan dipotong oleh sebuah transversal t seperti yang terlihat pada gambar di bawah. Jika ada tentukan:
a. Pasangan sudut sehadap.
=> tidak ada
b. Pasangan sudut dalam bersebrangan
=> tidak ada
c. Pasangan sudut luar bersebrangan
=> tidak ada.
d. Pasangan sudut sepihak
=> tidak ada.

Figure 3. Example of Students' Response (S1)

Following that, the authors invited students to describe their reasoning. Subject 1 (S1) responded that no pairs of angles are formed since the transversal cuts two lines that were not parallel (see Figure 4). Two more subjects indicated the same reason. It can be assumed that the undergraduate students understand that the two lines must be parallel in order to form pairs of angles.

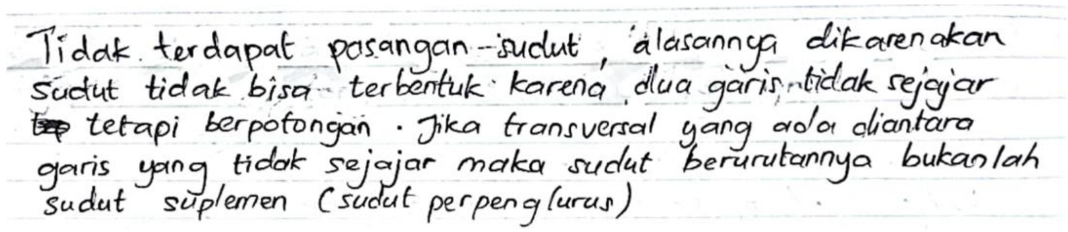

Tidak terdapat pasangan sudut, alasannya dikarenakan sudut tidak bisa terbentuk karena dua garis tidak sejajar tetapi berpotongan. Jika transversal yang ada diantara garis yang tidak sejajar maka sudut berurutannya bukanlah sudut suplemen (sudut perpeng lurus)

Figure 4. The example of reason (S1)

Two other subjects provided additional reasons. Figure 5 shows that Subject 3 (S3) answered that if any two nonparallel lines were cut by a transversal, then the corresponding angles were not congruent. Therefore, no pair of corresponding angles, alternate interior angles, alternate exterior angles, or same-side angles could be constructed. A similar reason was also expressed by another subject (S4). The subject answered that if line r was not parallel to t, then no pairs of angles could be formed since the measure of the angles was not congruent. It implies that the students think each pair of corresponding angles, alternate interior angles, and alternate exterior angles must be congruent.

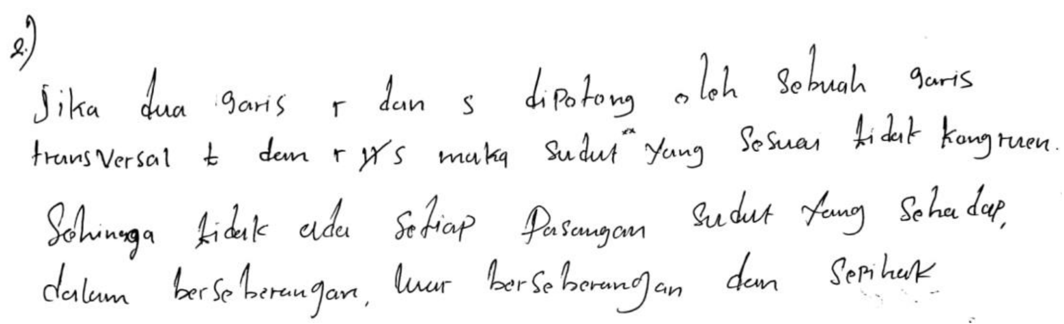

2.)
Jika dua garis r dan s dipotong oleh sebuah garis transversal t dan r ∦ s maka sudut" yang sesuai tidak kongruen.
Sehingga tidak ada setiap pasangan sudut yang sehadap, dalam bersebrangan, luar bersebrangan dan sepihak

Figure 5. Example of reason (S3)

One subject (S7) provided aadifferent answer. The subject answered questions 2.a, 2.b, and 2.c correctly. However, for 2.d, determining each pair of same-side angles, S7 provided an incorrect answer. Based on Figure 6, he opined that the pair of same-side angles could not be determined because of a lack of information regarding the measure of the given angles. This means that the condition for forming same-side angles requires knowledge of the measure of the angles.

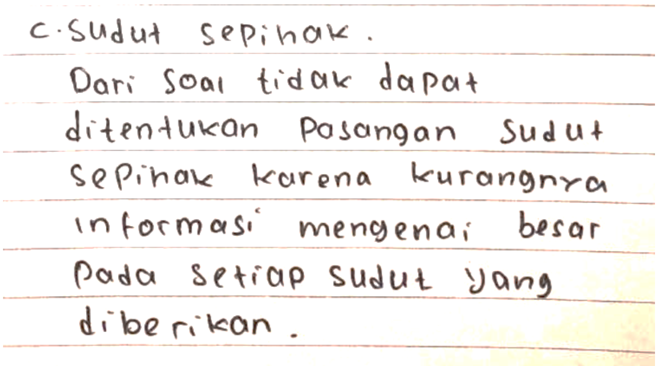


Figure 6. Example of reason (S7)

3.2 DISCUSSION

In the previous subsection, several findings about how the students' learned the knowledge of the concepts of angles formed by a transversal. From the results, the profile of the students' concept images can be highlighted. Some students have a good understanding of the concept. Unfortunately, many students have a gap between their concept images and the definition of the concept of angles formed by a transversal. The students' responses revealed three incorrect concepts were. The three concepts are: (1) the two lines cut by a transversal must be parallel in order to form special pairs of angles; (2) each pair of corresponding angles, alternate interior angles, and alternate exterior angles must be congruent; and (3) the condition for forming each pair of same-side angles must be known. According measure of the angles.

Refer to scholarly knowledge, the names of those angle pairs were first introduced with the condition that two non-parallel lines are cut by a transversal, where a pair of congruent angles will be formed only if the two lines are parallel. (Alexander & Koeberlein, 2020; Clemens et al., 1990; Leonard et al., 2014; Moise, 1990). So, two parallel lines are not a condition for forming a pair of corresponding angles, alternate interior angles, and alternate exterior angles. However, it is a condition for each pair of angles formed to be congruent. Alternatively, on the other hand as an implication if each pair of angles are parallel.

The concept images that were formed is, of course, different from the concept definition. Tall & Vinner (1981) stated that concept image is all cognitive structures in a person's mind that are tied to a specific topic. This may be nonsensical and may differ significantly from the formal concept definition. The concept gap, as explained before, might make it difficult for a person to understand and solve a problem relating to the notion (Sulastri et al., 2021). These concept images and gaps occur as a result of the subject's learning experience (Sulastri, 2023).

From the profile of students' concept images, it is obtained that the students are limited in understanding the concepts of angles formed by a transversal, where their understanding only revolves around parallel lines and congruent angles. It can be inferred that there is an obstacle known as an epistemological obstacle. Epistemological obstacle is a learning obstacle caused by limited context used when first introducing a concept (Brousseau, 1997). The result of this situation is that students will have difficulty using the concept when faced with a different context or may not even be able to apply it at all.

4. CONCLUSION

Didactic transposition is important to ensure that there are no gaps or misconceptions between student knowledge and scholarly knowledge. In the process of didactic transposition in mathematics learning, learned knowledge refers to the mathematics knowledge that students have acquired. From the learned knowledge, a profile of the students' concept images can be assessed. By revealing the concept image profile, any learning obstacles that arise can be identified. Research on the concept of angles formed by a transversal in undergraduate programs has revealed that students have diverse concept images. Many of them believe that a pair of angles, for example, an alternate interior angles, will be formed if two parallel lines are cut by a transversal. If the lines are non-parallel, the pairs of angles are not formed. Other gaps include the requirement for each pair of corresponding angles, alternate interior angles, and alternate exterior angles to be congruent, and the measure of the angles must be known, to form each pair of same-side angles. These results indicate that there was an epistemological learning obstacle. Based on these findings, further research is necessary to examine other steps of didactic transposition to achieve a comprehensive solution. Alternative learning designs can be developed to address these issues.

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